Channel Feedback for Reconfigurable Intelligent Surface Assisted Wireless Communications

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Abstract—**Reconfigurable intelligent surface (RIS) has recently received increasing interest due to the superiority of changing the wireless propagation environments intelligently. Channel feedback is essential in frequency division duplex (FDD) RIS-assisted wireless communications, since the downlink channel state information (CSI) has to be acquired by the base station (BS) for the joint beamforming at the BS and the RIS. In this paper, we first investigate the single-structured sparsity of RIS channel, which means that the angular-domain sparse channels of different users share the same non-zero element indices only in the column dimension, but not in the row dimension. Based on this channel characteristic, we then propose a dimension-reduced channel feedback scheme with a low overhead for channel feedback. Specifically, the downlink CSI can be decomposed into the structured CSI and unstructured CSI. The structured CSI of the multi-user channels can be fed back to the BS by only one user according to the single-structured sparsity, while the unstructured CSI can be fed back with a fairly low overhead by different users, respectively. Moreover, by utilizing the angle coherence time, the per-user overhead can be reduced further, while the near-optimal per-user rate can still be guaranteed.**

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) has been recently recognized as a promising technique for future 6G communications [1]. Instead of adapting to the propagation environments in existing wireless communication systems, RIS can change the propagation environment by leveraging its controllable metamaterial-based elements. To jointly design the beamforming at the base station (BS) and the RIS [2]–[4], it is essential for the BS to acquire the downlink channel state information (CSI). In the frequency division duplex (FDD) RIS-assisted wireless communications, we need to design a channel feedback scheme to feed the downlink CSI from the users back to the BS with the help of a channel codebook. Unfortunately, the size of the conventional codebook exponentially increases with the number of RIS elements and BS antennas [5], which results in a unbearable overhead for channel feedback in practice.

Although channel feedback has not been investigated for emerging RIS-assisted wireless communications, it has been widely studied in current 4G and 5G wireless communications [5], [6]. After channel estimation of the downlink CSI at the user, an appropriate codeword is selected by the user from a predetermined codebook to approximate the downlink CSI. Then, the index of this codeword is fed back to the BS via the uplink channel [6]. Based on the predetermined codebook and the received codeword index, the downlink CSI can be recovered at the BS. However, the existing channel feedback schemes cannot be directly applied to the RIS-assisted wireless communications, since the size of channel matrix becomes much larger (e.g., 32×64 for a single-antenna user [7]) than that of current wireless communications without RIS (e.g., 32×1 [5]), which will result in the considerable channel feedback overhead and very large codebook size (i.e., the number of codewords in the codebook). Besides, most of the existing codebooks consist of one-dimension vector codewords, which are not applicable for two-dimension channel in RIS-assisted wireless communications.

In this paper, channel feedback is investigated for the first time in RIS-assisted wireless communications to the best of our knowledge[1]. Firstly, we investigate the single-structured sparsity of RIS channel for RIS-assisted wireless communication systems. Specifically, the channel between the RIS and the user is *user-specific*, while the channel between the BS and the RIS is *user-independent*. In other words, different users share the same channels between the BS and the RIS, but have their respective channels between the RIS and users. We can find that the sparse angular-domain channels of different users share the same non-zero element indices only in the column dimension, but not in the row dimension. This characteristic is called as the single-structured sparsity in this paper. Then, a dimension-reduced channel feedback scheme for RIS-assisted wireless communications with a low overhead is proposed. Thanks to the single-structured sparsity, the downlink CSI can be decomposed into the structured CSI and unstructured CSI. The structured CSI of different users can be fed back to the BS only by one user, while the unstructured CSI can be fed back with a fairly low overhead by different users, respectively. Simulation results verify that the proposed scheme can obviously reduce the per-user channel feedback overhead.

Notation: $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ denote the transpose, conjugate transpose, and inversion of a matrix, respectively;

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[1] The simulation codes are provided to reproduce the results in this paper at http://oa.ee.tsinghua.edu.cn/dailinglong/publications/publications.html.

$\sin^2(\measuredangle(\mathbf{a},\mathbf{b})) = 1 - \frac{|\mathbf{a}^{\mathrm{H}}\mathbf{b}|^2}{\|\mathbf{a}\|^2\|\mathbf{b}\|^2}$; $\otimes$ denotes the kronecker product operator; $\mathbf{A}_{(:,n)}$ denotes the n-th column of the matrix $\mathbf{A}$. Finally, $\mathrm{E}[\cdot]$ denotes the expectation operator.

II. SYSTEM MODEL

In this paper, a narrowband RIS-assisted wireless communication system is considered as illustrated in Fig. 1, where a BS with M antennas is assisted by an RIS with N elements to simultaneously serve K single-antenna users. Hence, the downlink signal model for the k-th user can be expressed as [8]

$$y_k = \left(\mathbf{h}_{d,k}^T + \mathbf{h}_{r,k}^T\mathbf{\Phi}\mathbf{G}\right)\mathbf{x} + n_k, \tag{1}$$

where y_k is the received signal of the k-th user, $\mathbf{h}_{d,k}^T \in \mathbb{C}^{1\times M}$, $\mathbf{h}_{r,k}^T \in \mathbb{C}^{1\times N}$, and $\mathbf{G} \in \mathbb{C}^{N\times M}$ denote the direct BS-user channel from the BS to the k-th user, the RIS-user channel from the RIS to the k-th user, and the BS-RIS channel matrix, respectively; $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the precoded transmitted signal at the BS, and n_k is the complex Gaussian noise at the k-th user with zero mean and unit variance. Particularly, $\mathbf{\Phi} \in \mathbb{C}^{N\times N}$ represents the phase-shift diagonal matrix of the RIS as $\mathbf{\Phi} = \mathrm{diag}\left(\boldsymbol{\phi}^T\right) = \mathrm{diag}\left(\phi_1, \cdots, \phi_n, \cdots, \phi_N\right)$, where $\phi_n = e^{j\varphi_n}$ $(\varphi_n \in [0, 2\pi], n = 1, 2, \cdots, N)$ represents the phase shift of the n-th RIS element.

By utilizing the property of diagonal matrix, i.e., $\mathbf{h}_{r,k}^T\mathbf{\Phi}\mathbf{G} = \boldsymbol{\phi}^T\mathrm{diag}\left(\mathbf{h}_{r,k}^T\right)\mathbf{G}$, the equivalent baseband downlink channel $\mathbf{h}_{\mathrm{DL},k}^T \in \mathbb{C}^{1\times M}$ for the k-th user can be expressed as

$$\mathbf{h}_{\mathrm{DL},k}^T = \mathbf{h}_{d,k}^T + \mathbf{h}_{r,k}^T\mathbf{\Phi}\mathbf{G} = \mathbf{h}_{d,k}^T + \boldsymbol{\phi}^T\mathrm{diag}\left(\mathbf{h}_{r,k}^T\right)\mathbf{G}. \tag{2}$$

In this paper, we deploy a uniform linear array (ULA) of antennas at the BS and a uniform planar array (UPA) of elements at the RIS, respectively. Considering the classical narrowband ray-based channel model in this paper, the *user-independent* BS-RIS channel in spatial domain can be expressed as

$$\mathbf{G} = \sum_{i=1}^{L_1}\alpha_i\mathbf{b}_1\left(\phi_{1,i}, \theta_{1,i}\right)\mathbf{a}^H\left(\phi_i^{\mathrm{AoD}}\right), \tag{3}$$

where L_1 is the number of dominant paths between the BS and the RIS, α_i denotes the complex gain of the i-th path, and $\phi_{1,i}$ $(\theta_{1,i})$ denotes the azimuth (elevation) angle-of-arrival (AoA) of the i-th path, respectively. Considering a UPA with N_1 horizontal elements and N_2 vertical elements $(N = N_1 \times N_2)$, the steering vector $\mathbf{b}_1\left(\phi_{1,i}, \theta_{1,i}\right) \in \mathbb{C}^{N\times 1}$ of the i-th path can be expressed as

$$\mathbf{b}_1(\phi_{1,i},\theta_{1,i}) = \left[1, e^{j2\pi\frac{d_{\mathrm{R}}}{\lambda}\sin\theta_{1,i}}, \cdots, e^{j2\pi\frac{d_{\mathrm{R}}}{\lambda}(N_2-1)\sin\theta_{1,i}}\right]^{\mathrm{T}} \otimes \frac{1}{\sqrt{N}}\left[1, e^{j2\pi\frac{d_{\mathrm{R}}}{\lambda}\cos\theta_{1,i}\sin\phi_{1,i}}, \cdots, e^{j2\pi\frac{d_{\mathrm{R}}}{\lambda}(N_1-1)\cos\theta_{1,i}\sin\phi_{1,i}}\right]^{\mathrm{T}}, \tag{4}$$

where d_{R} is the element spacing at the RIS, λ is the wavelength of the carrier frequency.

The steering vector of the BS antenna array response of the i-th path $\mathbf{a}\left(\phi_i^{\mathrm{AoD}}\right) \in \mathbb{C}^{M\times 1}$ can be denoted as

$$\mathbf{a}(\phi_i^{\mathrm{AoD}}) = \frac{1}{\sqrt{M}}\left[1, e^{j2\pi\frac{d_{\mathrm{B}}}{\lambda}\sin\phi_i^{\mathrm{AoD}}}, \cdots, e^{j2\pi\frac{d_{\mathrm{B}}}{\lambda}(M-1)\sin\phi_i^{\mathrm{AoD}}}\right]^{\mathrm{T}}, \tag{5}$$

where d_{B} denotes the antenna spacing at the BS, and ϕ_i^{AoD} denotes the angle-of-departure (AoD) of the i-th path between the BS and the RIS.

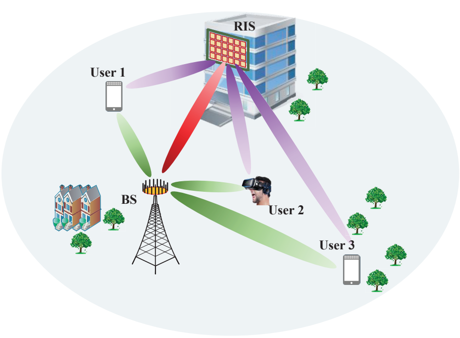

Fig. 1. RIS-assisted wireless communication system.

Similar to (3), the *user-specific* RIS-user channel $\mathbf{h}_{r,k}$ between the RIS and the k-th user in spatial domain can be expressed as

$$\mathbf{h}_{r,k}^T = \sum_{j=1}^{L_2}\beta_{k,j}\mathbf{b}_2^H\left(\phi_{2,k,j}, \theta_{2,k,j}\right), \tag{6}$$

where L_2 is the number of dominant paths between the RIS and the user, $\beta_{k,j}$ is the complex gain of the j-th path, $\phi_{2,k,j}$ and $\theta_{2,k,j}$ are the azimuth and elevation AoDs of the j-th path, respectively, and $\mathbf{b}_2\left(\phi_{2,k,j}, \theta_{2,k,j}\right)$ has the similar form with $\mathbf{b}_1\left(\phi_{1,i}, \theta_{1,i}\right)$ in (4).

According to (2), (3) and (6), we denote the cascaded channel [8] of the k-th user in spatial domain as $\mathbf{H}_k \triangleq \mathrm{diag}\left(\mathbf{h}_{r,k}^T\right)\mathbf{G}$, which can be expressed as

$$\mathbf{H}_k = \sum_{i=1}^{L_1}\sum_{j=1}^{L_2}\alpha_i\beta_{k,j}\mathrm{diag}\left(\mathbf{b}_2^H\left(\phi_{2,k,j}, \theta_{2,k,j}\right)\right) \mathbf{b}_1\left(\phi_{1,i}, \theta_{1,i}\right)\mathbf{a}^H\left(\phi_i^{\mathrm{AoD}}\right). \tag{7}$$

For simplicity, we rewrite (7) as

$$\mathbf{H}_k = \sum_{i=1}^{L_1}\sum_{j=1}^{L_2}g_{i,k,j}\mathbf{b}\left(\phi_{i,k,j}^{\mathrm{AoA}}, \theta_{i,k,j}^{\mathrm{AoA}}\right)\mathbf{a}^H\left(\phi_i^{\mathrm{AoD}}\right), \tag{8}$$

where $g_{i,k,j} \triangleq \alpha_i\beta_{k,j}$ is the complex channel gain for cascaded channel.

From (4) and (7), we can obtain the cascaded steering vector $\mathbf{b}\left(\phi_{i,k,j}^{\mathrm{AoA}}, \theta_{i,k,j}^{\mathrm{AoA}}\right) \triangleq \mathrm{diag}\left(\mathbf{b}_2^H\left(\phi_{2,k,j}, \theta_{2,k,j}\right)\right)\mathbf{b}_1\left(\phi_{1,i}, \theta_{1,i}\right)$, where $\phi_{i,k,j}^{\mathrm{AoA}}$ denotes the cascaded azimuth AoA at the RIS, and $\theta_{i,k,j}^{\mathrm{AoA}}$ denotes the cascaded elevation AoA. Note that $\phi_{i,k,j}^{\mathrm{AoA}}$ and $\theta_{i,k,j}^{\mathrm{AoA}}$ can be calculated according to the azimuth and elevation AoAs and AoDs at the RIS. In this paper, the $(\phi_{i,k,j}^{\mathrm{AoA}}, \theta_{i,k,j}^{\mathrm{AoA}})$ is termed as a pair of cascaded AoAs.

III. PROPOSED DIMENSION-REDUCED CHANNEL FEEDBACK SCHEME

In this section, the single-structured sparsity of the cascaded channel is introduced at first, based on which we then propose a dimension-reduced channel feedback scheme.

A. *Single-Structured Sparsity of the Cascaded Channel*

Sparsity of cascaded channel for one user: In RIS-assisted wireless communication systems, the BS and the RIS are usually surrounded by limited scatterers. In other words, there are only a few AoDs at the BS and a few cascaded AoAs at the RIS. Hence, we can rewrite the spatial domain channel $\mathbf{H}_k$ of the size $N \times M$ as [9]

$$\mathbf{H}_k = \tilde{\mathbf{H}}_k \mathbf{\Theta}_T^H = \mathbf{\Theta}_R \mathbf{A}_k \mathbf{\Theta}_T^H, \tag{9}$$

where $\mathbf{\Theta}_R \in \mathbb{C}^{N \times G_r}$ and $\mathbf{\Theta}_T \in \mathbb{C}^{M \times G_t}$ are the dictionary matrices for the angular-domain channel with angular resolutions G_r of the cascaded AoA at the RIS and G_t of the AoD at the BS, based on which we can divide the range of angles onto G_r and G_t grids, respectively. And $\mathbf{A}_k$ is the angular-domain channel, which is well studied in the existing researches.

In this paper, we denote $\tilde{\mathbf{H}}_k$ as the hybrid-domain (hybrid spatial- and angular-domain) channel, since $\tilde{\mathbf{H}}_k$ in the column dimension is converted into the angular domain by $\mathbf{\Theta}_T$, while $\tilde{\mathbf{H}}_k$ in the row dimension is still in the spatial domain. As can be seen from Fig. 2, there are only L_1 non-zero columns in hybrid domain channel $\tilde{\mathbf{H}}_k$, since there are L_1 AoDs at the BS. Besides, the indices of the non-zero column positions are corresponding to the grid indices of the AoDs.

To reduce the overhead for channel feedback, we can focus on feeding the non-zero columns of $\tilde{\mathbf{H}}_k$ back to the BS. According to (8) and (9), the i-th non-zero of $\tilde{\mathbf{H}}_k$, which is also the g_t-th column of $\tilde{\mathbf{H}}_k$ can be esxpressed as

$$\tilde{\mathbf{h}}_{k,i} \triangleq \tilde{\mathbf{H}}_{k,(:,g_t)} = \sum_{j=1}^{L_2} g_{i,k,j} \mathbf{b}\left(\phi_{i,k,j}^{\mathrm{AoA}}, \theta_{i,k,j}^{\mathrm{AoA}}\right), \tag{10}$$

where $g_t = 1, \cdots, G_t$ and $i = 1, \cdots, L_1$. Note that the relationship between g_t and i is not constant, since the grid index of the AoD, which determines the value of g_t, is time-varying. Besides, (10) can be rewrited as $\tilde{\mathbf{h}}_{k,i} = \mathbf{B}_{k,i}\mathbf{g}_{k,i}$, where the steering matrix $\mathbf{B}_{k,i} = \left[\mathbf{b}\left(\phi_{i,k,1}^{\mathrm{AoA}}, \theta_{i,k,1}^{\mathrm{AoA}}\right), \cdots, \mathbf{b}\left(\phi_{i,k,L_2}^{\mathrm{AoA}}, \theta_{i,k,L_2}^{\mathrm{AoA}}\right)\right] \in \mathbb{C}^{N \times L_2}$ and $\mathbf{g}_{k,i} = \left[g_{i,k,1}, \cdots, g_{i,k,L_2}\right]^T \in \mathbb{C}^{L_2 \times 1}$. There is a brief explanation for (10): Most of columns in $\mathbf{\Theta}_T$ are orthogonal to the steering vector $\mathbf{a}\left(\phi_i^{\mathrm{AoD}}\right)$, except when the grid angle of column vector is equal to the angle of steering vector.

Single-structured sparsity for different users: The cascaded channel consists of two parts: the *user-independent* BS-RIS channel and *user-specific* RIS-user channel, i.e., different users share the same channel between the BS and RIS, but have their respective channels between the RIS and users. In other words, different users have the same AoDs but respective cascaded AoAs, i.e., different users share the same non-zero columns in the hybrid domain. To sum up, for the cascaded channels of different users, the sparsity is structured only in the column dimension but not in the row dimension, which is termed as single-structured sparsity in this paper[2].

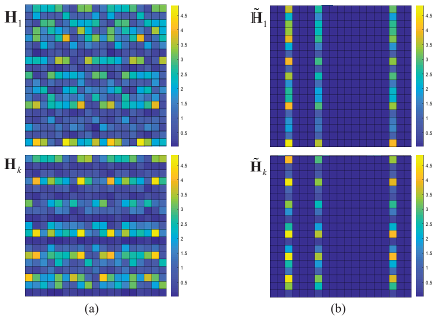

Fig. 2. The cascaded channels (a) in the spatial domain and (b) in the hybrid domain.

As shown in Fig. 2(b), the number of non-zero columns are limited, and different users' non-zero columns have the same indices, i.e., the sparsity is structured for different users in the column dimension, since we transform the user-independent AoDs with dictionary matrix. However, if we transform the user-specific AoAs with another dictionary matrix, we will acquire a few non-zero rows, but the indices of non-zero rows differ from different users, i.e., the structured sparsity appears only in the single side.

B. *Proposed Dimension-Reduced Channel Feedback*

In this subsection, we propose a dimension-reduced channel feedback scheme with three steps by utilizing the single-structured sparsity. We focus on the cascaded channel feedback in this paper[3]. According to Subsection III-A, we can just feed the non-zero columns of hybrid-domain channel $\tilde{\mathbf{H}}_k$. Specifically, what we need to feed back at the user are the non-zero column vectors and their position indices in the column dimension.

As shown in Fig. 3, the non-zero column indices are fed back to the BS in Step 1. We can just select one user to feed back these indices for all users according to the single-structured sparsity, which can significantly reduce the per-user channel feedback overhead. Then, the non-zero column vectors are fed back via Step 2 and Step 3. In Step 2, for every non-zero column vector, we design a dynamic codebook. Compared with the current schemes with a per-determined and constant codebook, the dynamic codebook can be better applied for time-varying channel, since it contains the time-varying angles of channels, which will be introduced later. To update the dynamic codebooks between the BS and the user, the angle information needs to feed back to the BS from the user. In Step 3, for every non-zero column vector, a codeword is selected from the corresponding dynamic codebook to approximate this non-zero vector. Then, this codeword index is fed back to the BS. And the same operations are carried out for the other non-zero columns.

To be more specific, we describe the three steps as follows:

Step 1: Feedback for the user-independent indices of non-zero columns. In this step, the indices of the non-zero

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[2] This channel characteristic has been recently used to improve the channel estimation accuracy in [9].

[3] Since there is no difference between the direct BS-user channel in RIS-assisted scneario and the conventional wireless channel in existing scneario.

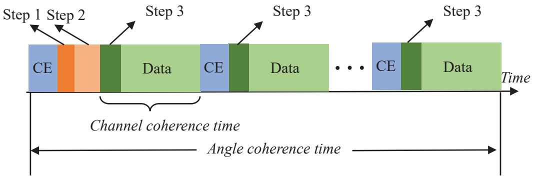

Fig. 3. Frame structure for the proposed dimension-reduced channel feedback. The downlink CSI should be acquired by channel estimation (CE) at the user firstly. Like other channel feedback research, we assume that the user can acquire downlink CSI perfectly [5]. The user-independent column indices are fed back in Step 1. The user-specific cascaded AoAs are fed back in Step 2. The codeword indices for channel vectors are fed back in Step 3.

column vectors of $\tilde{\mathbf{H}}_k$ will be fed back to the BS. According to (9), the angular resolution of the dictionary matrix $\mathbf{\Theta}_T$ is G_t, i.e., the range of AoDs are divided onto G_t grids. And the index of the AoD grid is also the position index of the non-zero column in $\tilde{\mathbf{H}}_k$. Thanks to the single-structured sparsity, we just need to appoint one user to feed the L_1 user-independent column indices (i.e., the grid indices of AoDs) back to the BS, while other users can skip this procedure. Hence, the channel feedback overhead can be significantly reduced from the following two aspects: 1) The number of users who carry out this step can be reduced from K to only 1 by utilizing the single-structured sparsity. 2) The number of indices which need to be fed back, can be reduced from the number of antennas M to the dominant paths L_1, where we usually have $L_1 \ll M$.

In the next two steps, non-zero column vectors are fed back to the BS one by one according to the corresponding dynamic codebooks.

Step 2: Feedback for the user-specific cascaded AoAs. To feed back the hybrid-domain channel $\tilde{\mathbf{H}}_k$, for every non-zero column, we design a dynamic codebook at the user with the help of cascaded AoAs at first. Then, to make sure the consistency of the codebook both at the BS and the user, we need to feed the cascaded AoAs back to the BS, which is different from the most traditional channel feedback schemes, since the codebooks of those schemes are constant.

Specifically, taking the i-th non-zero column vector $\tilde{\mathbf{h}}_{k,i}$ of $\tilde{\mathbf{H}}_k$ as an example, we design the dynamic codebook as follows. We utilize the subspace property of the non-zero column $\tilde{\mathbf{h}}_{k,i}$. Note that the N-dimensional non-zero channel vector composes by only L_2 paths between the RIS and the k-th user as shown in (10). In other words, $\tilde{\mathbf{h}}_{k,i}$ is distributed in the L_2-dimensional subspace of the N-dimensional channel space [5]. Hence, we introduce the steering matrix $\hat{\mathbf{B}}_{k,i}$, which composes by L_2 steering vectors corresponding to L_2 pair quantized cascaded AoAs $\{(\hat{\phi}_{i,k,j}^{\mathrm{AoA}}, \hat{\theta}_{i,k,j}^{\mathrm{AoA}})\}_{j=1}^{L_2}$ of $\tilde{\mathbf{h}}_{k,i}$, to help design the dynamic codebook. Note that the quantized cascaded AoA can be acquired by quantizing the cascaded AoA with B_0 bits. The q-th codeword of the dynamic codebook $\mathcal{C}_{k,i}$ can be expressed as

$$\mathbf{c}_{k,i,q} = \hat{\mathbf{B}}_{k,i}\mathbf{r}_{k,i,q}, \tag{11}$$

where $\mathbf{r}_{k,i,q} \in \mathbb{C}^{L_2 \times 1}$ is a low-dimensional codeword vector chosen from the conventional random vector quantization (RVQ) codebook [5] of size 2^B, and $\hat{\mathbf{B}}_{k,i} \in \mathbb{C}^{N \times L_2}$ is the steering matrix which can be expressed as

$$\hat{\mathbf{B}}_{k,i} = \left[\mathbf{b}\left(\hat{\phi}_{i,k,1}^{\mathrm{AoA}}, \hat{\theta}_{i,k,1}^{\mathrm{AoA}}\right), \cdots, \mathbf{b}\left(\hat{\phi}_{i,k,L_2}^{\mathrm{AoA}}, \hat{\theta}_{i,k,L_2}^{\mathrm{AoA}}\right)\right]. \tag{12}$$

Hence, we can acquire the dynamic codebook $\mathcal{C}_{k,i}$ with the size as 2^B ($q = 1, \cdots, 2^B$).

Besides, since the channel angles change slowly, the first two steps only need to be carried out once in every angle coherence time, which is much larger than the channel coherence time.

Step 3: Feedback for the proposed codebook indices. In this step, we select appropriate codewords to approximate the non-zero column vectors from the corresponding dynamic codebooks, respectively. Then, what we need to feed back are the indices of the codewords for those non-zero column vectors. Specifically, taking $\tilde{\mathbf{h}}_{k,i}$ as an example, we select the codeword with index $D_{k,i}$ from codebook $\mathcal{C}_{k,i}$, which satisfies the objective function as $D_{k,i} = \underset{q \in [1, 2^B]}{\arg\min} \sin^2\left(\measuredangle\left(\tilde{\mathbf{h}}_{k,i}, \mathbf{c}_{k,i,q}\right)\right)$. Then, this index will be fed back to the BS with B bits. Similarly, the codeword indices corresponding to other $L_1 - 1$ non-zero columns can be fed back in the same way.

According to the feedback parameters of the above three steps, the BS can recover the cascaded channel $\tilde{\mathbf{H}}_k$ as follows. According to Step 1, the BS can acquire the indices of the non-zero column vectors of all users. In Step 2 and Step 3, we can design dynamic codebooks and select the codewords from the corresponding codebooks to express the non-zero column vectors. Then, we can arrange those channel vectors into the corresponding positions to reconstruct the hybrid-domain channel $\tilde{\mathbf{H}}_k$. Finally, the channel $\mathbf{H}_k$ in the spatial domain can be calculated according to (9).

IV. SIMULATION RESULTS

In this section for simulations, the number of BS antennas, RIS elements, and single-antenna users are set as $M = 32$, $N = 64$, and $K = 4$, respectively. The number of paths are set as $L_1 = 4$ and $L_2 = 2$, respectively. The SNR at receiver is set as 5 dB.

After acquiring the downlink CSI via channel feedback, cross entropy optimization (CEO) [10] is utilized to optimize the joint beamforming at the BS and the RIS. We calculate the per-user rate to evaluate the performance of the channel feedback scheme, which can be expressed as $R = \mathrm{E}\left[\log_2\left(1 + \frac{\frac{\gamma}{K}\left|\mathbf{h}_{\mathrm{DL},k}^H \hat{\mathbf{v}}_k\right|^2}{1 + \frac{\gamma}{K}\sum_{i=1, i\neq k}^{K}\left|\mathbf{h}_{\mathrm{DL},k}^H \hat{\mathbf{v}}_i\right|^2}\right)\right]$, where γ is the transmit power, and $\hat{\mathbf{v}}_k \in \mathbb{C}^{M \times 1}$ is the k-th column of the normalized precoding matrix at the BS. The phase-shift diagonal matrix $\mathbf{\Phi}$ for the RIS has been considered in $\mathbf{h}_{\mathrm{DL},k}^H$, as shown in (2). The per-user overhead is the total feedback overhead divided by the number of users.

Fig. 4 shows the per-user rate versus the per-user feedback overhead. We compare the performance of the proposed scheme with a conventional channel statistics-based feedback scheme [5]. And we also provide the upper bound with the

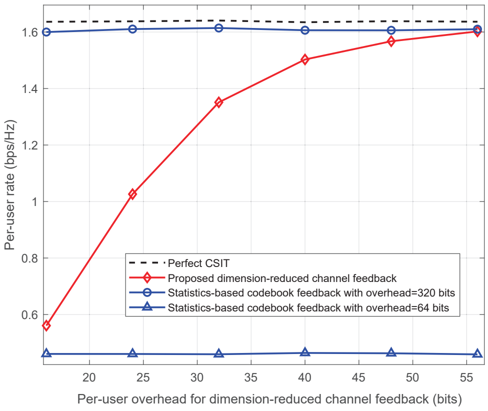

Fig. 4. The per-user rate comparison between the dimension-reduced channel feedback scheme and the conventional feedback scheme.

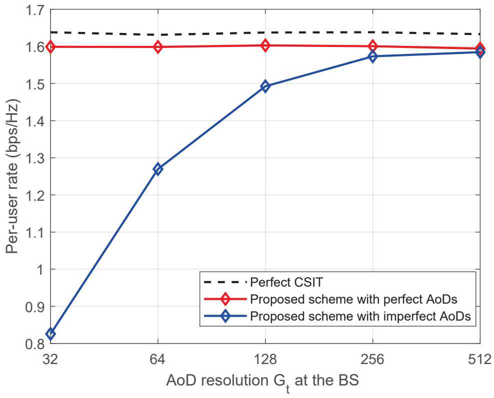

Fig. 5. The per-user rate of the dimension-reduced channel feedback scheme against the AoD resolution G_t at the BS.

perfect channel state information at the transmitter (CSIT). Here, we set the AoD resolution as $G_t = 512$ and the quantized bits of the cascaded AoA as $B_0 = 7$ bits. The ratio between the angle coherence time and the channel coherence time is set as $T_0 = 10$ [5]. The per-user overhead increases when the codebook size B grows from 1 to 11 bits. When the per-user overhead is $\frac{\log_2(G_t)\times L_1}{T_0 K} + \frac{B_0\times 2\times L_1\times L_2}{T_0} + BL_1 = 56$ bits with codebook size $B = 11$ bits, the proposed scheme can achieve the near-optimal performance. However, to achieve the same performance, the per-user overhead of the conventional scheme is 320 bits, which means the proposed scheme can reduce the overhead more than 80% while the performance can still be guaranteed.

Fig. 5 illustrates the impacts of the AoD resolution G_t for the proposed scheme, with $B = 11$ bits and $B_0 = 7$ bits. To feed the single-structured indices back to the BS in Step 1, AoD at the BS must be approximated to the nearest grid, which will result in angle deviations and bring imperfect AoDs. Increasing the number of grid will reduce the angle deviations, but introduce more overhead for channel feedback. From Fig. 5 we can find that when $G_t = 512$, the performance with imperfect AoDs on grid can almost attain the per-user rate of the proposed scheme with perfect AoDs. Meanwhile, the per-user overhead for Step 1 is pretty small thanks to the single-structured sparsity, since only one user needs to carry out this step. For example, the per-user feedback overhead of Step 1 is $\lceil L_1 \times \log_2(G_t)/(T_0 K)\rceil = 1$ bit.

V. CONCLUSIONS

In this paper, we investigated the channel feedback problem in RIS-assisted wireless communications for the first time to reduce the channel feedback overhead to the best of our knowledge. Firstly, we exploited the single-structured sparsity of the cascaded channel. Then, by utilizing this property, a dimension-reduced channel feedback scheme consisting of three steps was proposed with the help of dynamic codebooks. Besides, by leveraging the angle coherence time, the overhead can be reduced further. Simulation results show that the dimension-reduced channel feedback scheme can reduce the channel feedback overhead dramatically, while the near-optimal per-user rate can be guaranteed.

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